Chapter I

INTRODUCTION

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The assessment of the mechanical integrity of any flawed mechanical structure requires the development of approaches and methodologies which can deal not only with simple situations, such as small-scale yielding (SSY) under pure mode I isothermal loading, but also with much more complex situations, including large scale plasticity, mixed-mode cracking, and non-isothermal loading. Two types of approaches have been developed for this purpose, as schematically depicted on figure I.1.

The first approach, referred to as the “global” approach is essentially based on the extensive development over the past few decades of linear elastic fracture mechanics (LEFM) and then no linear mechanics (NLFM). In this approach which was historically the first one to be developed, it is assumed that the fracture resistance can be measured in terms of a single parameter, such as K_{Ic}, J_{Ic} or crack tip opening displacement (CTOD). Rules and standards uniquely based on the mechanical conditions of test conditions have been established for “valid” fracture toughness measurements, without paying attention to the failure micromechanisms. These rules are summarized in chapter IV. This global approach is extremely useful and absolutely necessary, but has also a number of limitations, in particular when large scale yielding conditions are met or when non isothermal conditions are encountered. Another limitation is the size effect which is usually observed when structural ferritic steels are tested in the brittle domain and in the ductile-to-brittle transition regime. It is now well established that fracture toughness is specimen size dependent even in the lower shelf regime of the ductile-to-brittle transition. The specimen size requirements in an NLFM test to measure a “valid” fracture toughness, J_{Ic} is also problematic. This raises the important problem related to the transferability of laboratory test results to large components tested under in-service conditions.

The second approach, called the “local approach” was developed later in the 80’s, although it can be considered that the models proposed by McClintock (McClintock, 1963; Mc Clintock, 1968) were already local approaches devoted to fatigue failure. In this methodology the modeling of fracture toughness is based on local fracture criteria established most often from tests on volume elements, in particular notched specimens. Then these criteria are applied to the crack-tip situation.The development of this methodology requires that, at least, two conditions are fulfilled, as illustrated in two conferences devoted to this topic (NED 1987 ; Euromech-Mecamat 1996): (i) Micromechanistically based models must be established; (ii) A perfect knowledge of the stress-strain field ahead of a stationary and a propagating crack is required. This has been made possible, thanks to the advent of analytical and numerical solutions.

In particular, the rapid development of finite element methods (FEM) has largely contributed to the advance of this methodology.

There has been considerable research on "local approaches" to fracture over the past two decades. These approaches either assume conventional material behavior supplemented by models of local failure processes, which is the situation for brittle fracture (see chapter V), or use continuum damage mechanics (CDM) involving coupled constitutive equations with a softening effect due to damage, which is the case for ductile fracture, as explained in detail in chapters VII and VIII. In this book a state of the art of the so-called Local Approach to Fracture is presented. Actually, the global and the local approaches to fracture are more complementary than contradictory. In particular the local approach to fracture, which necessitates detailed numerical FEM calculations, must be essentially used to deal with "difficult" situations involving complex loading conditions.

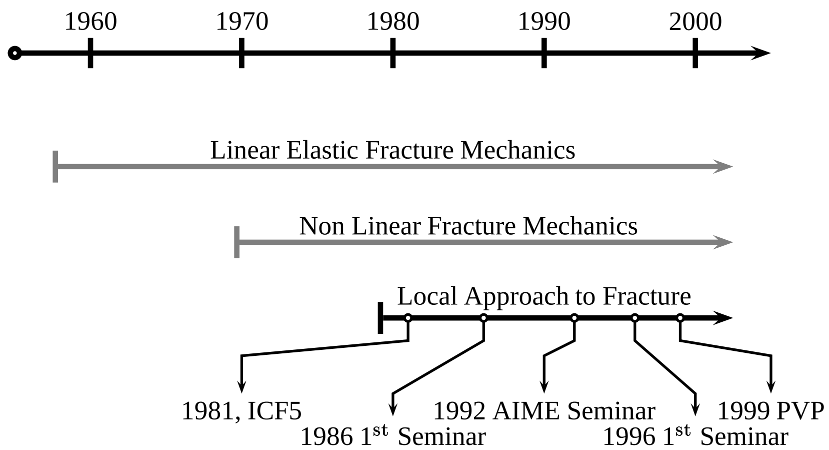

Figure I.1: Historical landmarks for the Local Approach to Fracture

1 GLOBAL APPROACH TO FRACTURE

1.1 SMALL SCALE YIELDING — LARGE SCALE YIELDING

The distinction between small scale yielding (SSY) and large scale yielding (LSY) is important with respect to the modeling of fracture (figure I.2). SSY corresponds to case where the size of the plastic zone is much smaller than the size of the specimen or structure. In that case the overall behavior of the structure remains linear. LSY corresponds to the opposite case: a large portion of the specimen is plastically yielded and the overall behavior is non linear. SSY can be analyzed using linear elastic fracture mechanics (see 1.2); LSY is analyzed using NLFM (see 1.3). Most of the testing standards require the specimens to remain under SSY to be valid. These requirements are however more and more difficult to meet as materials fracture and

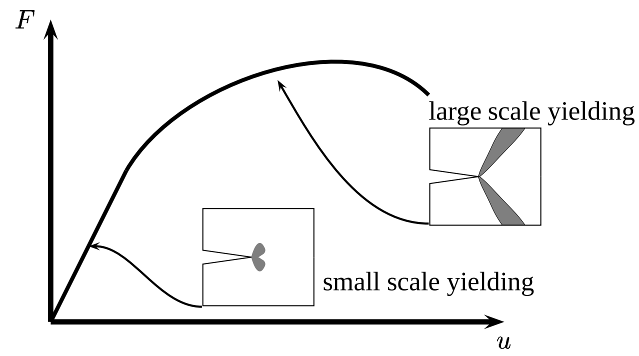

Figure I.2: Overall behavior of a cracked structure. u: displacement, F: force. The plastic zone is schematically shown in gray.

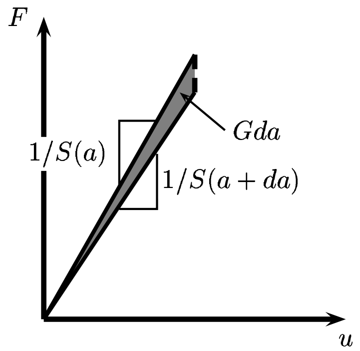

Figure I.3: Energy release rate in the linear case.

toughness properties are continuously improved. Using the "local approach to fracture" is particularly interesting in the case of LSY.

1.2 LINEAR ELASTIC FRACTURE MECHANICS (LEFM)

In the case of a structure whose global response is linear, the link between the global behavior and crack advance can be obtained using the variation of the stiffness or compliance, S, of the structure. This variation is assumed to be caused by crack extension only. In that case the variation of the stored energy corresponds to the energy dissipated by cracking. The stored elastic energy is given by: $E = \frac{1}{2}u^2/S$ where u is the prescribed displacement. Considering a constant value for u (figure I.3) the energy released by the structure, δE, for an increase of the crack surface, δA is equal to:

$$\delta E = \frac{1}{2}\frac{u^2}{K^2}\frac{dS}{dA}\delta A = G\delta A \tag{1}$$

G corresponds to the energy release rate.

The value of G can be related to the applied loading using the expression of the stress tensor close to the crack tip. Under mixed mode loading (I, II and III) stresses

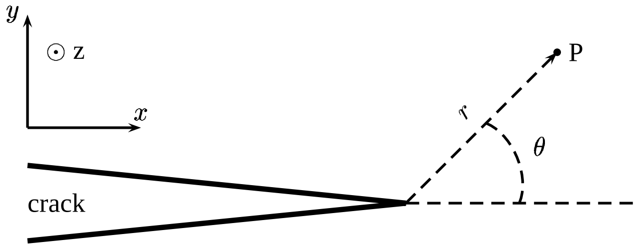

Figure I.4: Local coordinate system at the crack tip.

(figure I.4) are given by:

$$\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) - \frac{K_{II}}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\left(2+\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right) \quad (2)$$

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \quad (3)$$

$$+\frac{K_{II}}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2} \quad (4)$$

$$\sigma_{xy} = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\sin\frac{\theta}{2}\cos\frac{3\theta}{2} + \frac{K_{II}}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \quad (5)$$

$$\sigma_{xz} = -\frac{K_{III}}{\sqrt{2\pi r}}\sin\frac{\theta}{2} \quad (6)$$

$$\sigma_{yz} = \frac{K_{III}}{\sqrt{2\pi r}}\cos\frac{\theta}{2} \quad (7)$$

$$\sigma_{zz} = \begin{cases} \nu(\sigma_{xx}+\sigma_{yy}) & \text{plane strain} \\ 0 & \text{plane stress} \end{cases} \quad (8)$$

Displacements are given by:

$$u_x = \frac{K_I}{2\mu}\sqrt{\frac{r}{2\pi}}\cos\frac{\theta}{2}\left(\kappa-1+2\sin^2\frac{\theta}{2}\right) + \frac{K_{II}}{2\mu}\sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2}\left(\kappa+1+2\cos^2\frac{\theta}{2}\right) \quad (9)$$

$$u_y = \frac{K_I}{2\mu}\sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2}\left(\kappa+1-2\cos^2\frac{\theta}{2}\right) - \frac{K_{II}}{2\mu}\sqrt{\frac{r}{2\pi}}\cos\frac{\theta}{2}\left(\kappa-1-2\sin^2\frac{\theta}{2}\right) \quad (10)$$

$$u_z = 2\frac{K_{III}}{\mu}\sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2} \quad (11)$$

$$\kappa = \begin{cases} 3-4\nu & \text{plane strain} \\ (3-\nu)/(1+\nu) & \text{plane stress} \end{cases} \quad (12)$$

The corresponding energy release rate is given by:

$$G = \begin{cases} \dfrac{1-\nu^2}{E}\left(K_I^2 + K_{II}^2\right) + \dfrac{1+\nu}{E} K_{III}^2 & \text{plane strain} \\ \dfrac{1}{E}\left(K_I^2 + K_{II}^2\right) + \dfrac{1+\nu}{E} K_{III}^2 & \text{plane stress} \end{cases} \tag{13}$$

It may be useful to express the stresses in the cylindrical reference frame (the K_{III} terms remain unchanged):

$$\sigma_{rr} = \frac{\cos\frac{\theta}{2}}{\sqrt{2\pi r}}\left(K_I\left(1+\sin^2\frac{\theta}{2}\right) + 2K_{II}\left(3\cos\theta - 1\right)\tan\frac{\theta}{2}\right) \tag{14}$$

$$\sigma_{\theta\theta} = \frac{\cos\frac{\theta}{2}}{\sqrt{2\pi r}}\left(K_I\cos^2\frac{\theta}{2} - \frac{3}{2}K_{II}\sin\theta\right) \tag{15}$$

$$\sigma_{zz} = \frac{\cos\frac{\theta}{2}}{\sqrt{2\pi r}}\left(K_I\sin\theta + K_{II}\left(3\cos\theta - 1\right)\right) \tag{16}$$

The crack can propagate if the released energy equals the energy corresponding to the creation of two free surfaces ; that is $G = G_c = 2\gamma_s$, where γ_s is the surface energy. G_c corresponds to the critical energy release rate which is a material characteristic. The corresponding mode I plane strain fracture toughness is computed using equation 13:

$$K_{Ic} = \sqrt{\frac{EG_c}{1-\nu^2}} \tag{17}$$

Typical values for γ_s are of the order of magnitude of 1 J/m^2 which corresponds to a fracture toughness $K_{Ic} = 0.68$ MPa$\sqrt{\text{m}}$ (with $E = 210$ GPa and $\nu = 0.3$). This value is indeed very small and using equation 17 is only valid for very brittle materials (e.g. glass, ice, ...). In practice the actual value for G_c is (much) larger than $2\gamma_s$. In many materials, dissipation at the crack tip takes place in a given volume (so called process zone). This dissipation process originates from micro-cracking or phase transformation in though ceramics or ceramic matrix composites. In metals, dissipation corresponds to plasticity around the crack tip.

The above calculation of G, based on the compliance variation with crack advance (equation 1), is valid for elastic materials only. As mentioned above plasticity occurs at the crack tip. As long as small scale yielding prevails, it is possible to apply Irwin plastic zone correction (Irwin, 1957). Using this correction a crack of length a in an elastoplastic medium is considered as equivalent to an effective crack of length $a_e = a + r_y$ in a purely elastic medium. r_y is given by, under plane stress conditions:

$$r_y = \frac{1}{2\pi}\left(\frac{K_I}{\sigma_y}\right)^2 \tag{18}$$

where σ_y is the yield strength of the material. Using this model, the plastic zone size, R_p, is equal to $2r_y$. The safety analysis can then be performed as for an elastic medium using a_e instead of a. It should also be added that under plane strain conditions the plastic zone size is three times smaller than that corresponding to plane stress conditions, i.e. $R_p = (1/3\pi)(K_I/\sigma_y)^2$.